Extension of the Gelfond-Lifschitz Reduction for Preferred Answer Sets

Alexander Šimko
simko@fmph.uniba.sk

Department of Applied Informatics
Faculty of Mathematics, Physics and Informatics
Comenius University In Bratislava, Slovakia

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- If a slope is too difficult for a user, do not recommend it.
- If a user likes a slope, recommed it.
- If there is no snow on a slope, do no recommend it.

Recommend or not to recommend?

- the first rule is the weakest one,
- the third rule is the strongest one.

Do not recommend

$\mathcal{P} = (P, <)$

$$
\begin{array}{llll}
r_1: & \neg rec & \leftarrow & difficult, not\ rec \\
r_2: & rec & \leftarrow & likes, not\ \neg rec \\
r_3: & \neg rec & \leftarrow & no\_snow, not\ rec
\end{array}
$$

$$r_1 < r_2 < r_3$$

Answer Sets

$\{rec, \ldots\}$, $\{\neg rec, \ldots\}$

How should semantics change in the presence of preferences on rules?

Select the subset of the standard answer sets as preferred.

Preferred Answer Sets

$\{\neg rec, \ldots\}$

Existing approaches, e.g.:

- Brewka and Eiter, Delgrande et al., Wang et al.,
- Zhang and Foo, Sakama and Inoue, Šefránek

Independent rules: $\{a, b\}$

$r_1 : a \leftarrow$
$r_2 : b \leftarrow$

Exception: $\{b\}$

$r_1 : a \leftarrow not\ b$
$r_2 : b \leftarrow$

Conflicting rules: $\{a\}$, $\{b\}$

$r_1 : a \leftarrow not\ b$
$r_2 : b \leftarrow not\ a$

Preference handling as the reverse transformation

conflicts $\rightarrow$ exceptions?

Remove default negated literals from a preferred conflicting rule

$\mathcal{P}$		$t(\mathcal{P})$
$r_1 : a \leftarrow not\ b$	$\rightarrow$	$r_1 : a \leftarrow$
$r_2 : b \leftarrow not\ a$		$r_2 : b \leftarrow not\ a$

$r_2 < r_1$

And define $\mathcal{PAS}(\mathcal{P}) = \mathcal{AS}(t(\mathcal{P}))$

- How the transformation looks like?
- What is the direct definition of the semantics?
- What are the properties of the semantics?
- What is the connection with existing approaches?

A rule is an expression of the from

$$l_0 \leftarrow l_1, \ldots, l_m, not\ l_{m+1}, \ldots, not\ l_n,$$

$$head(r) = l_0,\ body^+(r) = \{l_1, \ldots, l_m\},\ body^-(r) = \{l_{m+1}, \ldots, l_n\}$$

An answer set of a program P without *not* is given by the bottom-up evaluation using $T_P(X) = \{head(r) : body^+(r) \subseteq X\}$ from $\emptyset$.

$r_1 : a \leftarrow$	$X_0 = \emptyset$
$r_2 : b \leftarrow a$	$X_1 = \{a\}$
$r_3 : d \leftarrow c$	$X_2 = \{a, b\}$
	$X_3 = X_2$

Answer sets of programs with *not* are defined using Gelfond-Lifschitz reduction:

For a program P and a set of literals S we obtain P^S by:

- removing each rule r with $body^-(r) \cap S \neq \emptyset$, and
- removing *not* from the remaining rules.

Set of literal S is an answer set of a program P iff

$$S \text{ is answer set of } P^S$$

Two rules are conflicting if they are of the form

$$a \leftarrow \ldots, \mathit{not}\ b$$
$$b \leftarrow \ldots, \mathit{not}\ a$$

Simple case – Each head has different head:

Remove from the body of a rule the head of a less preferred conflicting rule.

$$\begin{array}{lcl} r_1 : a \leftarrow not\ b & & a \leftarrow \\ r_2 : b \leftarrow not\ a & \rightarrow & b \leftarrow not\ a \end{array}$$

$r_2 < r_1$

This is not usable in general:

$$\begin{array}{lcl} r_1 : a \leftarrow x, not\ b & & a \leftarrow x \\ r_2 : b \leftarrow y, not\ a & \rightarrow & b \leftarrow y \\ r_3 : a \leftarrow z, not\ b & & a \leftarrow z, not\ b \end{array}$$

$r_3 < r_2 < r_1$

In the body of r_2 we need to distinguish between "a" derived by r_1 and r_3.

Solution:

- Introduce special-purpose literals n_r,
- divide each rule r into rules:
  - deriving n_r,
  - deriving $head(r)$,
- replace default negated literals by n_r literals

$$
\begin{array}{lll}
r_1 : a \leftarrow x, not\ b & & n_{r_1} \leftarrow x, not\ n_{r_2} & & n_{r_1} \leftarrow x \\
 & & a \leftarrow n_{r_1} & & a \leftarrow n_{r_1} \\
r_2 : b \leftarrow y, not\ a & \rightarrow & n_{r_2} \leftarrow y, not\ n_{r_1}, not\ n_{r_3} & \rightarrow & n_{r_2} \leftarrow y, not\ n_{r_1} \\
 & & b \leftarrow n_{r_2} & & b \leftarrow n_{r_2} \\
r_3 : a \leftarrow z, not\ b & & n_{r_3} \leftarrow z, not\ n_{r_2} & & n_{r_3} \leftarrow z, not\ n_{r_2} \\
 & & a \leftarrow n_{r_3} & & a \leftarrow n_{r_3}
\end{array}
$$

$r_3 < r_2 < r_1$

An answer set S can be represented by the rules that generate it:

$$\Gamma_P(S) = \{r \in P : body^+(r) \subseteq S \text{ and } body^-(r) \cap S = \emptyset\}$$

An answer set X is preferred iff for each $r \in P \setminus \Gamma_P(X)$:

- $body^+(r) \not\subseteq X$, or
- $body^-(r) \cap \{head(t) : t \in \Gamma_P(X)$ and t is not less preferred conflicting with r$\} \neq \emptyset$.

$\mathcal{P} = (P, <)$.

- Compatible with the answer set semantics:
  - $\mathcal{PAS}(\mathcal{P}) \subseteq \mathcal{AS}(P)$,
  - If $<= \emptyset$ or P is stratified, then $\mathcal{PAS}(\mathcal{P}) = \mathcal{AS}(P)$
- Brewka and Eiter's Principle I and II are satisfied.

- Deciding whether a $\mathcal{PAS}(\mathcal{P}) \neq \emptyset$ is NP-complete.
- Semantics does not guarantee existence of a preferred answer set when a standard one exits:

$r_1 : a \leftarrow not\ b$
$r_2 : b \leftarrow not\ a$

$r_3 : inc \leftarrow a, not\ inc$

$r_2 < r_1$

- If P is call-consistent and head-consistent (no integrity constraints via default and explicit negation), then

$$\mathcal{PAS}(\mathcal{P}) \neq \emptyset \text{ if } \mathcal{AS}(P) \neq \emptyset$$

Schaub and Wang: $\mathcal{PAS}_{DST}(\mathcal{P}) \subseteq \mathcal{PAS}_{WZL}(\mathcal{P}) \subseteq \mathcal{PAS}_{BE}(\mathcal{P})$

We: $\mathcal{PAS}_{BE}(\mathcal{P}) \subseteq \mathcal{PAS}(\mathcal{P})$

An answer set X of P is a *BE preferred answer set* of $\mathcal{P}$ iff there is an enumeration $\langle r_i \rangle$ of $\Gamma_P(X)$ such that for each i, j:

1. if $r_i < r_j$, then $j < i$, and
2. if $r_i < r$ and $r \in P \setminus \Gamma_P(X)$, then
   1. $body^+(r) \not\subseteq X$ or
   2. $body^-(r) \cap \{head(r_j) : j < i\} \neq \emptyset$ or
   3. $head(r) \in X$

An answer set X is preferred iff for each $r \in P \setminus \Gamma_P(X)$:

- $body^+(r) \not\subseteq X$, or
- $body^-(r) \cap \{head(t) : t \in \Gamma_P(X)$ and t is not less preferred conflicting with r$\} \neq \emptyset$.

- The semantics is not prescriptive
- The semantics is equivalent with answer set semantics for stratified programs
- Ignores preferences between non-conflicting rule, suiltable when preferences are automatically generated.

- Restriction to direct conflicts were made for two reasons:
  - It is good to proceed from simple cases to complex ones,
  - It was necessary in order to obtain the result

$$\mathcal{PAS}_{BE}(\mathcal{P}) \subseteq \mathcal{PAS}(\mathcal{P})$$

- Plan to extend the semantics to indirect conflicts