Extension of the Gelfond-Lifschitz Reduction for Preferred Answer Sets

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If a slope is too difficult for a user, do not recommend it.
If a user likes a slope, recommend it.
If there is no snow on a slope, do not recommend it.

Recommend or not to recommend?

- the first rule is the weakest one,
- the third rule is the strongest one.

Do not recommend
The Quetion of the paper
Preliminaries
Transformation
A Direct Definition of the Semantics
Properties
Comparison to Other Approaches
Where to Go From Here?

\[ \mathcal{P} = (P, <) \]

\begin{align*}
  r_1 : & \quad \neg rec \quad \leftarrow \quad \text{difficult, not rec} \\
  r_2 : & \quad rec \quad \leftarrow \quad \text{likes, not } \neg rec \\
  r_3 : & \quad \neg rec \quad \leftarrow \quad \text{no\_snow, not rec} \\
\end{align*}

\[ r_1 < r_2 < r_3 \]

Answer Sets
\{ rec, \ldots \}, \{ \neg rec, \ldots \}
How should semantics change in the presence of preferences on rules?

Select the subset of the standard answer sets as preferred.

Preferred Answer Sets

\{\neg rec, \ldots \}
Existing approaches, e.g.:

- Brewka and Eiter, Delgrande et al., Wang et al.,
- Zhang and Foo, Sakama and Inoue, Šefránek
Independent rules: \{a, b\}

\[ r_1 : a \leftarrow \]
\[ r_2 : b \leftarrow \]

Exception: \{b\}

\[ r_1 : a \leftarrow \text{not } b \]
\[ r_2 : b \leftarrow \]

Conflicting rules: \{a\}, \{b\}

\[ r_1 : a \leftarrow \text{not } b \]
\[ r_2 : b \leftarrow \text{not } a \]
Preference handling as the reverse transformation

conflicts $\rightarrow$ exceptions?

Remove default negated literals from a preferred conflicting rule

\[ \mathcal{P} \]

\[ r_1 : a \leftarrow \text{not} \; b \quad \rightarrow \quad r_1 : a \leftarrow \]

\[ r_2 : b \leftarrow \text{not} \; a \quad \rightarrow \quad r_2 : b \leftarrow \text{not} \; a \]

\[ r_2 < r_1 \]

And define \( \mathcal{PAS}(\mathcal{P}) = \mathcal{AS}(t(\mathcal{P})) \)
How the transformation looks like?
What is the direct definition of the semantics?
What are the properties of the semantics?
What is the connection with existing approaches?
A rule is an expression of the form

\[ l_0 \leftarrow l_1, \ldots, l_m, \text{not } l_{m+1}, \ldots, \text{not } l_n, \]

\[ \text{head}(r) = l_0, \ \text{body}^+(r) = \{l_1, \ldots, l_m\}, \ \text{body}^-(r) = \{l_{m+1}, \ldots, l_n\} \]
An answer set of a program $P$ without \textit{not} is given by the bottom-up evaluation using $T_P(X) = \{\text{head}(r) : \text{body}^+(r) \subseteq X\}$ from $\emptyset$.

\[\begin{align*}
  r_1 : a & \leftarrow \\
  r_2 : b & \leftarrow a & X_0 = \emptyset \\
  r_3 : d & \leftarrow c & X_1 = \{a\} \\
  & & X_2 = \{a, b\} \\
  & & X_3 = X_2
\end{align*}\]
Answer sets of programs with *not* are defined using Gelfond-Lifschitz reduction:

For a program $P$ and a set of literals $S$ we obtain $P^S$ by:
- removing each rule $r$ with $\text{body}^-(r) \cap S \neq \emptyset$, and
- removing *not* from the remaining rules.
Set of literal $S$ is an answer set of a program $P$ iff

\[ S \text{ is answer set of } P^S \]
Two rules are conflicting if they are of the form

\[ a \leftarrow \ldots, \textit{not } b \]

\[ b \leftarrow \ldots, \textit{not } a \]
Simple case – Each head has different head:

Remove from the body of a rule the head of a less preferred conflicting rule.

\[ r_1 : a \leftarrow not b \quad \quad a \leftarrow \]
\[ r_2 : b \leftarrow not a \quad \rightarrow \quad b \leftarrow not a \]

\[ r_2 < r_1 \]
This is not usable in general:

\[ r_1 : a \leftarrow x, \text{not } b \quad \quad a \leftarrow x \]
\[ r_2 : b \leftarrow y, \text{not } a \quad \rightarrow \quad b \leftarrow y \]
\[ r_3 : a \leftarrow z, \text{not } b \quad \quad a \leftarrow z, \text{not } b \]

\[ r_3 < r_2 < r_1 \]

In the body of \( r_2 \) we need to distinguish between "\( a \)" derived by \( r_1 \) and \( r_3 \).
Solution:

- Introduce special-purpose literals $n_r$,
- divide each rule $r$ into rules:
  - deriving $n_r$,
  - deriving $\text{head}(r)$,
- replace default negated literals by $n_r$ literals
$r_1 : a \leftarrow x, \text{not } b$

$n_{r_1} \leftarrow x, \text{not } n_{r_2}$

$a \leftarrow n_{r_1}$

$r_2 : b \leftarrow y, \text{not } a$

$n_{r_2} \leftarrow y, \text{not } n_{r_1}, \text{not } n_{r_3}$

$b \leftarrow n_{r_2}$

$r_3 : a \leftarrow z, \text{not } b$

$n_{r_3} \leftarrow z, \text{not } n_{r_2}$

$a \leftarrow n_{r_3}$

$r_3 < r_2 < r_1$

$n_{r_1} \leftarrow x$

$a \leftarrow n_{r_1}$

$n_{r_2} \leftarrow y, \text{not } n_{r_1}$

$b \leftarrow n_{r_2}$

$n_{r_3} \leftarrow z, \text{not } n_{r_2}$

$a \leftarrow n_{r_3}$
An answer set $S$ can be represented by the rules that generate it:

$$\Gamma_P(S) = \{ r \in P : body^+(r) \subseteq S \text{ and } body^-(r) \cap S = \emptyset \}$$
An answer set $X$ is preferred iff for each $r \in P \setminus \Gamma_P(X)$:

- $body^+(r) \not\subseteq X$, or
- $body^-(r) \cap \{head(t) : t \in \Gamma_P(X) \text{ and } t \text{ is not less preferred conflicting with } r\} \neq \emptyset$. 

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$\mathcal{P} = (P, <)$.

- Compatible with the answer set semantics:
  - $\mathcal{PAS}(\mathcal{P}) \subseteq AS(P)$,
  - If $<=$ $\emptyset$ or $P$ is stratified, then $\mathcal{PAS}(\mathcal{P}) = AS(P)$

- Brewka and Eiter’s Principle I and II are satisfied.
Deciding whether a $\mathcal{PAS}(\mathcal{P}) \neq \emptyset$ is NP-complete.

Semantics does not guarantee existence of a preferred answer set when a standard one exits:

$r_1 : a \leftarrow \text{not } b$

$r_2 : b \leftarrow \text{not } a$

$r_3 : \text{inc} \leftarrow a, \text{not } \text{inc}$

$r_2 < r_1$
If \( P \) is call-consistent and head-consistent (no integrity constraints via default and explicit negation), then

\[
\mathcal{PAS}(P) \neq \emptyset \text{ if } \mathcal{AS}(P) \neq \emptyset
\]
Schaub and Wang: $\mathcal{PAS}_{DST}(\mathcal{P}) \subseteq \mathcal{PAS}_{WZL}(\mathcal{P}) \subseteq \mathcal{PAS}_{BE}(\mathcal{P})$

We: $\mathcal{PAS}_{BE}(\mathcal{P}) \subseteq \mathcal{PAS}(\mathcal{P})$
An answer set $X$ of $P$ is a *BE preferred answer set* of $P$ iff there is an enumeration $\langle r_i \rangle$ of $\Gamma_P(X)$ such that for each $i, j$:

1. if $r_i < r_j$, then $j < i$, and
2. if $r_i < r$ and $r \in P \setminus \Gamma_P(X)$, then
   1. $body^+(r) \not\subseteq X$ or
   2. $body^-(r) \cap \{head(r_j) : j < i\} \neq \emptyset$ or
   3. $head(r) \in X$
An answer set $X$ is preferred iff for each $r \in P \setminus \Gamma_P(X)$:

- $body^+(r) \not\subseteq X$, or
- $body^-(r) \cap \{head(t) : t \in \Gamma_P(X) \text{ and } t \text{ is not less preferred conflicting with } r\} \neq \emptyset$. 
• The semantics is not prescriptive
• The semantics is equivalent with answer set semantics for stratified programs
• Ignores preferences between non-conflicting rule, suitable when preferences are automatically generated.
Restriction to direct conflicts were made for two reasons:
- It is good to proceed from simple cases to complex ones,
- It was necessary in order to obtain the result

\[ \mathcal{PAS}_{BE}(\mathcal{P}) \subseteq \mathcal{PAS}(\mathcal{P}) \]

- Plan to extend the semantics to indirect conflicts